A Note on Nāṭyaśāstra's *Śruti*-system: An Attempt to Reconstruct Ancient Indian Temperament

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In Nāṭyaśāstra, Bharata illustrates the śruti-system by four cycle tuning after discussing about consonant relation between two notes and the difference between Ṣaḍja-grāma and Madhyama-grāma. His description is as follows:

> *dve vīṇe tulyapramāṇatantryupavādanadaṇḍamūrcchane kṛtvā ṣaḍja-grāmāśrite kārye. tayor anyatarasyāṃ pañcamasyāpakarṣe śrutiṃ madhyamagrāmikīṃ kṛtvā tām eva ca pañcamasya śrutyutkarṣa-vaśāt ṣaḍjagrāmikīṃ kuryāt. ekaśrutir apakṛṣṭā bhavati. punar api tadvad evāpakarṣet, yathā gāndhāraniṣādavantāv itarasyāṃ ṛsabhadhaivatau pravekṣyataḥ dviśrutyadhikatvāt punar api tad-vad evāpakṛṣṭāyāṃ dhaivatārṣabhāv itarasyāṃ pañcamaṣaḍjau pra-vekṣyataḥ triśrutyadhikatvāt. tadvat punar apakṛṣṭāyāṃ pañcama-madhyamaṣaḍjā itarasyāṃ madhyamagāndhāraniṣādavantaḥ pra-vekṣyanti catuḥśrutyabhyadhikatvāt. evam etena śrutinidarśane-na dvaigrāmikyaḥ dvāviṃśatiśrutayaḥ pratyavagantavyāḥ*[1]

The above passage usually have been translated in the following manner.

> Having made two *vīṇās* equal in measure, string, *upavādana* (the place where the strings are tied up at the lower end of the *vīṇās*), rod and tuning, both of them should be made (tuned) in depen-dance on *Ṣaḍja-grāma*. On one of these, having made the *śruti* belong to *Madhyama-grāma* on account of lowering of *Pañcama*,

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one should make the same (*śruti*) belong to *Ṣaḍja-grāma* on account of raising of *Pañcama* (relatively). (Then) one *śruti* is lowered. <u>Again one should lower in the same manner so that (the strings of) *Gāndhāra* and *Niṣāda* (on the changed *vīṇā*) will enter the *Ṛṣabha* and *Dhaivata* on the other (fixed *vīṇā*), because of their being higher by two *śrutis*.</u> When (the *vīṇā*) is lowered in the same manner again, the *Ṛṣabha* and *Dhaivata* will enter the *Pañcama* and *Ṣaḍja* on the other (fixed *vīṇā*), because of their being higher by three *śrutis*. When (the *vīṇā*) is lowered in the same manner again, *Pañcama*, *Madhyama* and *Ṣaḍja* will enter (the strings of) *Madhyama*, *Gāndhāra* and *Niṣāda*, because of their being higher by four *śrutis*. Thus through this demonstration of *śruti*, twenty two *śrutis* belonging to two *grāma* should be understood[2].

Let us call the above interpretation "interpretation-A." This interpretation is supported by Abhinavagupta's famous commentary[3].

However, someone might translate the underlined passage into followings:

One should lower again in the very manner in which (the strings of) *Gāndhāra* and *Niṣāda* (on the changed *vīṇā*) will enter the *Ṛṣabha* and *Dhaivata* on the other (fixed *vīṇā*), because of their being higher by two *śrutis*(*punar api tadvad evāpakarṣet, yathā gāndhāraniṣādavantāv itarasyām ṛṣabhadhaivatau pravekṣyataḥ dviśrutyadhikatvāt*)[4].

From the Sanskrit syntactical point of view, there is some difficulty in the above translation[5]. In spite of such a textual difficulty, this interpretation

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[2]cf. Manomohan Ghosh's translation. Manomohan Ghosh: *The Nāṭyaśāstra*, 1st edition 1951, revised 3rd ed. Calcutta 1995, vol. 2, p. 6-7

[3]GOS vol. 4, p. 23: Explaining the underlined phrase, he says: In the changeable *vīṇā*, beginning with *Pañcama*, one should lower one *śruti* of all notes, where *Gāndhāra* and *Niṣāda* on changeable *vīṇā* will be lowered to the same position where *Ṛṣabha* and *Dhaivata* are placed on fixed *vīṇā*.(*calavīṇāyāṃ pañcamopakramaṃ srvasvarāṇām ekaśrutiṃ pātayet yeṣu calavīṇāgatau gāndhāraniṣādau dhruvavīṇāgatābhyāṃ dhaivatarṣabhābhyāṃ samasthānatayāpakarṣatvaṃ gacchataḥ.*)

[4]cf. Ācārya Bṛhaspati's interpretation: Nāṭyaśāstra Chapter 28: Ancient Scales of Indian Music with Sañjīvanam Commentary of Ācārya Bṛhaspati, introduced and translated by Bharat Gupt, Delhi 1995

[5]I acknowledge Prof. Diwakar Acharya's and Prof. Yuko Yokochi's advice from Sanskrit syntactical point of view. Especially Prof. Diwakar Acharya told me in his mail dated Feb

is fascinating, because it lets us diverge from 22 equal temperament, which seems to be unnatural. In addition we have no definitive text of Nāṭyaśāstra. Let us call this interpretation "interpretation-B".

The aim of this essay is to show a possible ancient(non medieval) Indian temperament on basis of the interpretation-B The difference between interpretation A and B will be demonstrated by Figure 1.

From Figure 1, you can easily see that the standard interval of each operation, which is the interval between Pa of Ṣaḍja-grāma and that of Madhyama-grāma and called *pramāṇa śruti*, is equal in the case of interpretation-A, and that the standard intervals of four step operations are not needed to be equal in the case of interpretation-B, In other words, on the supposition that 2nd and later tuning steps depend on the interval of certain notes on the fixed *vīṇā*, not on *pramāṇa śruti*, we could say that there are four kind of *śruti*-intervals corresponding to four tuning steps. On the other hand, on the supposition that each tuning depends only on the *pramāṇa śruti*, four kinds of *śruti*-interval take the same value.

In this situation, can we mathematically calculate the value of *śruti* or *śruti*s? In the case of interpretation-A, it is easy at least for us. One octave is divided into 22-*śruti*s equally, therefore its value is the quotient when dividing 1200 cents by 22, that is 54.545454..cents, or its ratio of frequency is $\sqrt[22]{2} = 1.03200827..$

On the other hand, it is not so easy to calculate the values of four *śruti*s in the case of interpretation-B. Considering other Bharata's descriptions of *śruti* and scale, however, I don't think it impossible.

Regarding the ratio of four kinds of *śruti*s, 9 *śruti*-interval and 13 *śruti*-interval, which are mentioned as consonant interval in Nāṭyaśāstra[6], must be taken into consideration. In a sense, *śruti*-system seems to be introduced for defining the consonant interval. On the position of interpretation-B, as a matter of course, these significant intervals can be measured by some combinations of four kinds of *śruti*-intervals.

In Nāṭyaśāstra, Sa-Pa, Ri-Dha, Ga-Ni and Sa-Ma are mentioned as consonant pairs(*saṃvāinau*) on *Ṣaḍja-grāma*. The former three are 13 *śruti*-interval, and the latter one is 9 *śruti*-interval. On the other hand, Ri-Dha, Ga-Ni, Sa-Ma and Ri-Pa are consonant pairs on *Madhyama-grāma*. The for-

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26th 2007 as follows: Apart from omission of *yathā* in one of the text readings, I would say *eva* is an obstacle between *tadvat* and *yathā*. If *tadvat ... yathā* were to mean "such a way that Niṣāda and Gandhāra ...", *eva* would not have come together with *tadvad*. Although GOS's new edition reports many variant reading concerning this passage, any reading where the *eva* in question is omitted is not found.
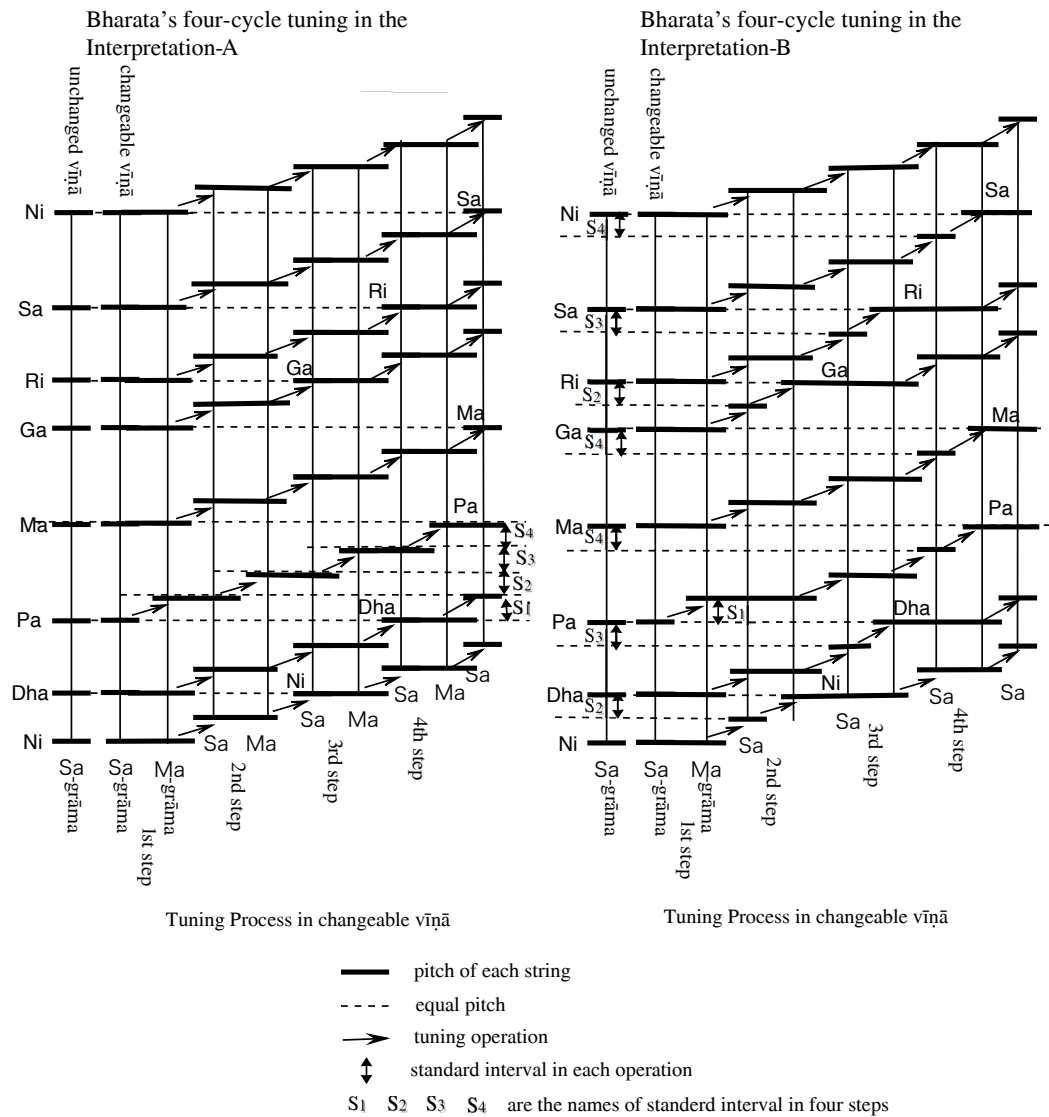
[6]Nāṭyaśāstra 28. 22f, GOS p. 14ff.

Figure 1: Two Alternative Tuning Systems

mer two are 13 *śruti*-interval, and the other two are 9 *śruti*-interval[7]. These relations can be illustrated by Table 1

Table 1: The Śruti and Consonant Relation

Notess		Śruti	Consonant Relation	
on Sa-grāma	on Ma-grāma		*13śruti*	*9śruti*
Ni	Ni			
		S_4		
K-Ni	K-Ni	S_3		
		S_2		
Sa	Sa	S_1		
		S_3		
		S_2		
Ri	Ri	S_1		
		S_2		
Ga	Ga	S_1		
		S_4		
A-Ga	A-Ga	S_3		
		S_2		
Ma	Ma	S_1		
		S_4		
		S_3		
	Pa	S_2		
Pa		S_1		
		S_3		
		S_2		
Dha	Dha	S_1		
		S_2		
Ni	Ni	S_1		

From Table 1, the following equations will follow.

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[7]Nāṭyaśāstra, prose after 28. 22 and 23, GOS vol. 4 p. 15 GOS: *yayoś ca navakatrayo-daśakam antaraṃ tāv anyonyaṃ saṃvādī. tad yathā ṣaḍjapañcamau, ṛṣabhadhaivatau gāndhāraniṣādavantau ṣaḍjamadhyamāv iti ṣaḍjagrāme. madhyamagrāme 'py evam eva. ṣaḍjapañcamarṣabhayoś cātra saṃvādaḥ. atra ślokaḥ –*
saṃvādo madhyamagrāme pañcamasyarṣabhasya ca/
ṣaḍjagrāme tu ṣaḍjasya saṃvādaḥ pañcamasya ca//

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$$n \;=\; 4S_1 + 4S_2 + 3S_3 + 2S_4 \tag{1}$$

$$m \;=\; 3S_1 + 3S_2 + 2S_3 + S_4 \tag{2}$$

$$m \;=\; 2S_1 + 3S_2 + 2S_3 + 2S_4 \tag{3}$$

In these equations, n and m stand for 13 *śruti*-intervals and 9 *śruti*-interval respectively. If these simultaneous linear equations are solved, we can define the values of *śruti*s, that is S_1, S_2, S_3 and S_4. However, the above equations can not be solved for n and m completely. Another relation of four kinds of *śruti*s is needed for any solution. Bharata's description of transposition seems to offer a key to solution.

Bharata describes the system of transposition after his explanation of two kinds of scales and the quasi-scales(*mūrcchanā*)[8]. In that portion, he says as follows: Taking *Gāndhāra* augmented by two *śruti* on *ṣaḍja-grāma* for *Dhaivata*, we can regard *Madhyama*, etc. as *Niṣāda*, etc. on *madhyama-grāma* in their order. On the other hand, through lowering of *Dhaivata* on *madhyama-grāma*, the name of notes will change because of the equality of *śruti*-intervals. That is, there is four *śruti*-interval between *Pñcama* and *Dhaivata* on *madhyama-grāma*, and augmenting of *Gāndhāra* makes it four *śruti*-interval. *Madhyama*, *Pañcama*, *Dhaivata*, etc. become *Niṣāda*, *Ṣaḍja*, *Ṛṣabha*, etc. on *ṣaḍja-grāma* respectively[9]. His explanation can be illustrated by Table 2.

In Table 2, let us compare the interval between Pa and Dha on *madhyama-grāma* with other four *śruti*-interval. The former consists of S_2, S_3 and two S_1, but the latter does of S_1, S_2, S_3 and S_4. The equality of both intervals is paralel with Equation 2 and 3. On the contrary, if the interval augmented by two *śruti*s is equal to the interval Dha and Ni on *madhyama-grāma*, we can get a new equation 4.

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[8]Nāṭyaśāstra, prose after Chapter 28, 33 GOS vol. 4 p. 26: *dvividhaiva mūrcha-nāsiddhiḥ tatra ṣaḍjagrāme dviśrutyutkarṣād dhaivatīkṛte gāndhāre mūrcchanāgrāmayor anyatvam. tadvaśān madhyamādayo yathāsaṃkhyena niṣāditvaṃ pratipadyante. tad-van madhyamagrāme dhaivatamārdavād dvaividhyam tulyaśrutyantaratvāca ca saṃjñā-nyatvam. madhyamagrāme hi catuḥśrutikam antaraṃ pañcamadhaivatayoḥ tadgāndhāro-tkarṣāc catuḥśrutikam eva bhavati. śeṣāś cāpi madhyamapañcamadhaivataniṣādaṣaḍja-rṣabhā niṣādaṣaḍjarṣabhagāndhāramadhyamapañcamatvaṃ prāpnuvanti tulyaśrutyantara-tvād antaranidarśanam api śrutinidarśane proktam.*

[9]Considering each *śruti*-interval between notes, I think this order of notes deviate from their own scale, because *Madhyama* does not correspond to *Niṣāda* but to *Ṣaḍja*.. Although I don't know what this deviation means, there does not seem to be any contradiction with other descriptions of Bharata on the supposition that *Madhyama* on *Ma-grāma* corresponds to *Ṣaḍja* on *Sa-grāma*.

Table 2: Bharata's System of Transposition

Sa-$grāma \Rightarrow Ma$-$grāma$					Ma-$grāma \Rightarrow Sa$-$grāma$				
Original			Transposed		Original			Transposed	
Notes	Śruti-Set1		Notes	Śruti-Set2	Notes	Śruti-Set3		Notes	Śruti-Set4
Sa	S_3	→	Ma	S_4	Ma	S_4	→	Sa	S_3
	S_2			S_3		S_3			S_2
Ri	S_1	→	Pa	S_2	Pa	S_2	→	Ri	S_1
	S_2			S_1		S_1			S_2
Ga	S_1			S_3		S_3		Ga	S_1
	S_4	↘		S_2		S_2	↗		S_4
	S_3		Dha	S_1	Dha	S_1			S_3
	S_2			S_2		S_2			S_2
Ma	S_1	→	Ni	S_1	Ni	S_1	→	Ma	S_1
	S_4			S_4		S_4			S_4
	S_3			S_3		S_3			S_3
	S_2			S_2		S_2			S_2
Pa	S_1	→	Sa	S_1	Sa	S_1	→	Pa	S_1
	S_3			S_3		S_3			S_3
	S_2			S_2		S_2			S_2
Dha	S_1	→	Ri	S_1	Ri	S_1	→	Dha	S_1
	S_2			S_2		S_2			S_2
Ni	S_1	→	Ga	S_1	Ga	S_1	→	Ni	S_1
	S_4			S_4		S_4			S_4
	S_3			S_3		S_3			S_3
	S_2			S_2		S_2			S_2
Sa	S_1	→	Ma	S_1	Ma	S_1	→	Sa	S_1

$$S_1 + S_2 \quad = \quad S_3 + S_4 \tag{4}$$

The evidence of Equation 4 is weaker than that of Equation 1, 2 and 3. Equation 4 is only supported by the fact that the augmented interval and that between Dha and Ni are both called "two-*śruti*." This fact, however, seems to me to be important. The interval called "two-*śruti*" is really used on musical scale in Bharata's period. In other words we can say that the musicians in Bharata's period knew how much interval is called "two-*śruti*," differing from the case of "one-*śruti*." On the contrary, if both "two-*śruti*-"intervals are not equal, can we define the values of *śruti*? I will mention this problem later.

On the supposition that both "two-*śruti*-"intervals are equal, we can solve the system of simultaneous liner equations, that is Equation 1, 2, 3 and 4. Its solution is as follows:

$$S_1, S_4 \quad = \quad \frac{5n - 7m}{2}$$
$$S_2, S_3 \quad = \quad 3m - 2n$$

Therefore, the values of 2-*śruti*, 3-*śruti* and 4-*śruti* intervals can be expressed in terms of the following equations.

$$2-\text{$śruti$}-\text{interval} \quad = \quad \frac{n - m}{2}$$
$$3-\text{$śruti$}-\text{interval} \quad = \quad \frac{5m - 3n}{2}$$
$$4-\text{$śruti$}-\text{interval} \quad = \quad n - m$$

In these equations, n and m stand for the values of 13-*śruti* and 9-*śruti*, intervals respectively. If every *śruti* are not equal, most of us would accept that 13-*śruti* and 9-*śruti* correspond to perfect forth($\frac{3}{2}$, 702cents), and perfect fifth($\frac{4}{3}$, 498cents). When we assign the values of perfect forth and fifth to m and n, we get the following equations.

$$2-\text{$śruti$}-\text{interval} \quad = \quad \frac{3}{2\sqrt{2}}, 101.96cents$$
$$3-\text{$śruti$}-\text{interval} \quad = \quad \frac{2^6\sqrt{2}}{3^4}, 192.18cents$$
$$4-\text{$śruti$}-\text{interval} \quad = \quad \frac{3^2}{2^3}, 203.91cents$$

Using the above equations, we can show a possible temperament coinciding with Bharata's tuning in Table 3

Table 3: A Possible Temperament

Ṣaḍja-grāma			Madhyama-grāma		
Notes	Ratio	Cents	Notes	Ratio	Cents
Sa	1	0	Ma	1	0
Ri	$\frac{2^6\sqrt{2}}{3^4}$	192.18	Pa	$\frac{2^6\sqrt{2}}{3^4}$	192.18
Ga	$\frac{2^5}{3^3}$	294.13	Dha	$\frac{2^3\sqrt{2}}{3^2}$	396.09
A-Ga	$\frac{2^4}{3^2\sqrt{2}}$	396.09	Ni	$\frac{2^2}{3}$	498.04
Ma	$\frac{2^2}{3}$	498.04	K-Ni	$\sqrt{2}$	600.0
Pa	$\frac{3}{2}$	701.96	Sa	$\frac{3}{2}$	701.96
Dha	$\frac{2^5\sqrt{2}}{3^3}$	894.13	Ri	$\frac{2^5\sqrt{2}}{3^3}$	894.13
Ni	$\frac{2^4}{3^2}$	996.09	Ga	$\frac{2^4}{3^2}$	996.09
K-Ni	$\frac{2^3}{3\sqrt{2}}$	1098.04	A-Ga	$\frac{2^3}{3\sqrt{2}}$	1098.04
Sa	2	1200	Ma	2	1200

This temperament differs from those proposed by preceding modern studies[10]. Is there no similarity to any studies? In my calculation Equation 4 is required. As I have already said, however, the evidence of Equation 4 is weaker than that of other required equations. If Equation 4 does not hold, we can assign semitone($\frac{16}{15}$, 112 cents), minor tone($\frac{10}{9}$, 182 cents) and major tone($\frac{9}{8}$, 204 cents) to 2-śruti, 3-śruti and 4-śruti intervals respectively, because these values satisfy Equation 1, 2 and 3 simultaneously. In this case, the value of pramāna-śruti, as Fox-Strangway said, is $\frac{81}{80} = 22$cents[11]. Other theories also are similar to this one at some points. However, as Lavy

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[10]Mark Levy: *Intonation in North Indian Music*, New Delhi 1981, pp 26 - 33
[11]lbid. pp 26-28

said, the theories of Fox-Strangway, Kolinski, Bake and Jairazbhoy agree that Bharata's 22 *śruti*s most likely did not arise from a conscious division of the octave[12]. As a matter of fact, Bharata's four cycle tuning can be performed on the supposition that a certain well-tuned scale have already been established. At this point it differs from the cases of ancient Greek or China.

As regards my calculation, there remain some problems to be discussed. The first problem concerns text. As far as my limited observations go, there is no reading which clearly supports the interpretation-B. In addition, the famous and influential commentary, Abhinavabhāratī clearly supports the interpretation-A. This fact, I think, must be examined. Widdess pointed out that the discrimination between 3-*śruti* and 4-*śruti* intervals had not functioned by the time of earliest sources of notated music, for example, Kuḍmiyāmalai inscription(A. D. 7 or 8 c.)[13]. His conclusion seems to me to be agreeable, and means that Bharata's 22-*śruti* system had been ruined by the time of Abhinavagupta (A.D. 11c.). That is to say it is very probable that Abhinavagupta did not know Bharata's four cycle tuning in the original sense, and that his interpretation reflects our reading of Nāṭyaśāstra[14].

The second problem is the meaning of my calculation. My calculation presupposes the possibility of interpretation-B and well-tuned scale on ancient Indian vīṇa. That is to say, on the presupposition that the interpretation-B is possible, we only say that the temperament represented in Table 3 can be inferred from Bharata's four cycle tuning. Even if the interpretation-B is possible, someone might say that Bharata's four cycle tuning only shows three sizes of scale steps without considering their ratio. In the case of interpretation-A, it would be a dominant opinion. As a matter of fact, Bake and Jairazbhoy think that the *śruti*s were considered by Bharata to have been equal in size, but Jairazbhoy believes that *śruti*s could not actually have been equal in practice[15]. This means that Bharata only shows three sizes of scale steps in actual sense, and that we cannot infer a specified temperament from Bharata's description. I have no evidence to reject such a opinion, and it seems to me to be most plausible for the present. Does it mean the meaninglessness of such calculations as I have done?

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[12] lbid. p 33

[13] Richard Widdess: *The Rāgas of Early Indian Music*, Oxford 1995, pp 205 - 246

[14] At the same time, Kavi's indication must be noticed. He pointed out that there are two recensions of text of Nāṭyaśāstra, and that one recension coincides with Abhinavagupta's commentary but another recension seems to be earlier than Abhinavagupta. *Nāṭyaśāstra of Bharatamuni* vol. 1 GOS 36, 4th edition 1992, Preface pp 65 - 68

[15] Mark Levy lbid. pp 30 - 31

I think the meanings of our calculation are two. One is to show an understanding of Bharata's four cycle tuning where perfect fourth and fifth can be adopted, and the other to examine the possibility of temperament that is consistent with Bharata's descriptions. As regards the first point, there are some difficulties in the reading of text, but the needed change of reading seems to be slight. In addition, it is a way of departing from 22 equal temperament, which does not seem to be plausible. On the other hand, an attempt to seek any temperament consistent with Bharata's description seems to be too technical, and its solution does not seem to be implausible. However, I think it would be a criterion for understanding of Bharata's temperament because of consistency with his description.

What we can safely say for the present, I think is only that there are three sizes in scale steps. As I have already said, Bharata does not describe the way of tuning but explain the characteristics of scale($gr\bar{a}ma$) and musical intervals by using the concept of $\acute{s}ruti$. In this context, we cannot deny the possibility that $\acute{s}ruti$ is only an ideal concept. Nevertheless I think this calculation will offer a criterion for the study of ancient Indian temperament. There are many obstacles in its studies, for example poverty of philological sources and the gap in Indian musical tradition. In such situations, if we examine any model of ancient Indian temperament, we must start with Bharata's description, and seek the consistency with it. From this point of view, my calculation would not meaningless.